Business PreCalculus MATH 1643 Section 004, Spring 2014

Lesson 12: Lines

In this lesson we study lines. Just as we measure weight and temperature by a number, we measure the "steepness" of a line by a number called its slope.

Definition 1. Slope of a Line: *The slope of a nonvertical line that passes through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is denoted by m and is defined by*

$$m = \frac{change\, in\, y - coordinates}{change\, in\, x - coordinates} = \frac{y_2 - y_1}{x_2 - x_1},\ x_1 \neq x_2.$$

The slope of a vertical line is undefined.

Example 1. *Find the slope of the line passing through $P(1, -1)$ and $Q(3, 3)$.*
Solution:

$$\begin{aligned} m &= \frac{change\, in\, y - coordinates}{change\, in\, x - coordinates} \\ m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - (-1)}{3 - 1} \\ &= \frac{4}{2} = 2 \end{aligned}$$

Definition 2. Point-Slope Form of the Equation of a Line: *If a line has slope m and passes through the point (x_1, y_1), then the* point-slope form *of an equation of the line is*

$$y - y_1 = m(x - x_1).$$

Example 2. *Find the equation of the line in* example 1.
Solution: *We know that the slope is $m = 2$, then we can use either $P(1, -1)$ or $Q(3, 3)$ to find the equation of the line. Hence*

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-1) &= 2(x - 1) \\ y + 1 &= 2x - 2 \\ y &= 2x - 3 \end{aligned}$$

Definition 3. Slope-Intercept Form of the Equation of a Line: *The* slope-intercept *of the equation of the line with slope m and y-intercept b is*

$$y = mx + b.$$

Definition 4. Horizontal and Vertical Lines:

1. *An equation of a horizontal line through (h, k) is $y = k$.*
2. *An equation of a vertical line through (h, k) is $x = h$.*

Definition 5. __**Standard Form of the Equation of a Line**__ *The equation*

$$ax + by + c = 0$$

is called the standard form *of the equation of a line. Note that* a, b, *and* c *are constants.*

Example 3. *Find the slope and* y*-intercept of the line with equation* $3x - 4y + 12 = 0$.
__**Solution:**__ *The goal is to rewrite the equation* $3x - 4y + 12 = 0$ *in the* slope-intercept:

$$\begin{aligned} 3x - 4y + 12 &= 0 \quad \textit{original equation} \\ 3x + 12 &= 4y \quad \textit{isolate } y \\ y &= \frac{1}{4}(3x + 12) \quad \textit{dividing by } 4 \\ y &= \frac{3}{4}x + 3 \quad \textit{simplify} \end{aligned}$$

Hence, the slope of the line of equation $3x - 4y + 12 = 0$ *is* $m = \frac{3}{4}$ *and the* y*-intercept is* $b = 3$.

Definition 6. __**Parallel and Perpendicular Lines:**__ *Let* L_1 *and* L_2 *be two distinct lines with slopes* m_1 *and* m_2*, respectively. Then*

$$L_1 \text{ is parallel to } L_2 \text{ if and only if } m_1 = m_2$$

$$L_1 \text{ is perpendicular to } L_2 \text{ if and only if } m_1 \cdot m_2 = -1$$

Example 4. *Let* $L : 2x - 3y + 6 = 0$ *be a straight line. Let* L_1 *and* L_2 *be two lines passing through the point* $(2, 8)$*. Let* L_1 *be parallel to* L *and* L_2 *be perpendicular to* L*. Find the equations of* L_1 *and* L_2.